Exam F (Part I) **Name**

Please Note: Present you solutions in a well organized, legible way. Calculators may be used *only* in elementary computational mode and trig mode, and not in calculus mode (even for exploratory purposes).

PART A. Each of the five multiple choice problems is worth 8 points. *Be sure to enter your answers on the second page of the test.*

1. The value of $\lim_{\Delta x \to 0} \dfrac{\cos(\frac{\pi}{6} + \Delta x) \; - \; \cos\frac{\pi}{6}}{\Delta x}$ is

a. $\cos(\frac{\pi}{6})$ **b.** $-\frac{1}{2}$ **c.** this $\frac{0}{0}$ limit does not exist **d.** $\frac{1}{2}$ **e.** $\frac{\sqrt{2}}{2}$

2. Consider the long sum

$$(1)^2 \cdot \tfrac{1}{1000} + (1 + \tfrac{1}{1000})^2 \cdot \tfrac{1}{1000} + (1 + \tfrac{2}{1000})^2 \cdot \tfrac{1}{1000} + (1 + \tfrac{3}{1000})^2 \cdot \tfrac{1}{1000} + \cdots + (1 + \tfrac{999}{1000})^2 \cdot \tfrac{1}{1000}\,.$$

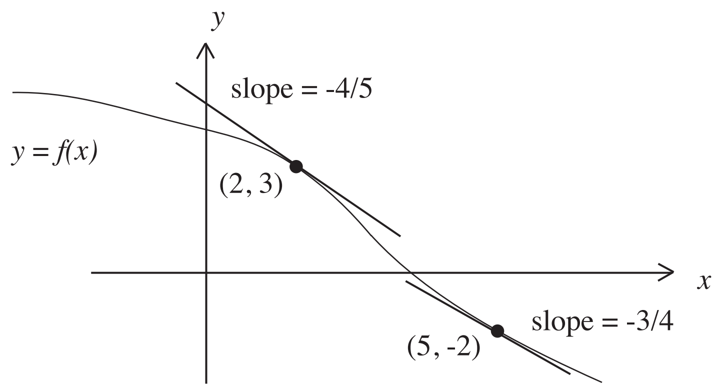
Determine a function $y = f(x)$ such that this long sum is very nearly equal to $\displaystyle\int_1^2 f(x)\,dx$. The approximate value of the long sum is:

a. 1 **b.** $\frac{1}{5}$ **c.** $\frac{11}{5}$ **d.** $\frac{5}{3}$ **e.** $\frac{7}{3}$.

3. Use logarithmic differentiation to find the derivative of $f(x) = x^x$ for $x > 0$. The value $f'(e)$ is equal to

a. $e(e^{e-1})$ **b.** $2e^e$ **c.** e^{-e} **d.** e **e.** $f(x)$ is not differentiable at $x = e$.

4. You are given a differentiable function $y = f(x)$. What is known about its graph is captured in the diagram below. Conclude from the information provided that the derivative of $y = f^{-1}(x)$



evaluated at $x = -2$ is:

a. $-\frac{3}{4}$ **b.** $-\frac{4}{3}$ **c.** $-\frac{1}{5}$ **d.** $-\frac{5}{4}$ **e.** f^{-1} may not be differentiable at $x = -2$.

5. Find a function $y = f(x)$ that satisfies $\frac{dy}{dx} = 6x^2(x^3+1)^{\frac{1}{2}}$ and $f(-1) = 0$. Then

a. $f(0) = \frac{1}{3}$ **b.** $f(0) = 1$ **c.** $f(0) = \frac{8}{3}$ **d.** $f(0) = \frac{4}{3}$ **e.** $f(0) = -\frac{7}{3}$.

Please enter your Answers for PART A.

1. ______ **2.** ______ **3.** ______ **4.** ______ **5.** ______

PART B.

1. Consider the function $f(x) = x(3x^2 - 15x)^{\frac{1}{3}}$.

i. Verify (show all steps) that $f'(x) = \dfrac{5x^2 - 20x}{(3x^2 - 15x)^{\frac{2}{3}}}$.

ii. (5 pts) Locate all the critical numbers for $y = f(x)$ on the number line below.

iii. Determine the interval(s) over which f is increasing and those over which f is decreasing, as well as the local maxima and minima of f.

2. Show that the surface area of the region obtained by rotating the graph of $f(x) = x^3$ one revolution around the x-axis is 64π square units.

3. Consider the differential equation $(1 + x)y' + y = \sqrt{x}$.

This equation fits the integrating factor format with $p(x) =$ ________________ and $q(x) =$ ________________ .

The integrating factor is $e^{P(x)} =$ ________________ .

Find a solution $y = f(x)$ of the differential equation that satisfies the condition $f(0) = -7$.

Formulas and Facts:

$F = ma \qquad \kappa = \frac{A_t}{t} \qquad c^2 = a^2 - b^2 \qquad c = \varepsilon a \qquad \kappa = \frac{ab\pi}{T}$

$F_P = C_P m \frac{1}{r^2} \qquad G = 6.67 \times 10^{-11}$ in M.K.S. $\qquad F = G\frac{mM}{r^2} \qquad F = \frac{8\kappa^2 m}{L}\frac{1}{r_P^2} \qquad \frac{a^3}{T^2} = \frac{GM}{4\pi^2}$.

$\frac{d}{dx}a^x = \ln a \cdot a^x \qquad \log_a x = \frac{1}{\ln a} \cdot \ln x$

$\frac{d}{dx}f^{-1}(x) = \frac{1}{f'(f^{-1}(x))} \qquad \sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{1}{2}$

$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1}(x) = \frac{1}{x^2+1}$

$$\pi\int_a^b f(x)^2\,dx \qquad \int_a^b \sqrt{1+f'(x)^2}\,dx \qquad 2\pi\int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$$